## Measurement and Density: Pre-lab

1) What is the goal of this lab? Summarize in a few sentences.
2) This lab requires you to use three different instruments to measure various lengths. How many decimal places should each measurement have for each instrument?
3) How will you use your height and diameter measurements to determine the volume of the cylinder?
4) How will you use your measurements to experimentally determine the density of the cylinder?

## Measurement and Density

## Objectives:

To learn how to perform basic measurements of length and mass using various instruments, to use such measurements in determining the density of an object and to practice error analysis.

## Introduction:

Everything from the nuclei in atoms to the countless stars in the universe contains mass and occupies some volume of space. Indeed it is the case that all types of matter possess both mass and volume.

Mass is an internal property of an object that seems to be very intuitive and natural for us, but, in actuality, it has no precise definition. The only constructive way to define mass is to relate it to the way in which it can be measured. In a typical measurement process, some property of a physical object is compared against the like in some standard object. For instance, to say a person is 2 meters tall is to suggest that if we were to connect two standard meter sticks end-to-end, the same distance will have been covered. It is of utmost important that we specify a measurement in terms of an appropriate set of units such that others can make sense of our findings. There are three known ways to define mass:

1. As a measure of inertia - the property of any physical object to resist any change in its state of motion, including changes to its speed and direction. This is sometimes referred to as inertial mass.
2. As a measure of the gravitational force between two objects. This is sometimes referred to as gravitational mass.
3. Energy-mass definition. As a measure of total energy contained within an object if this object is at rest, using famous Einstein equation $m=E / c^{2}$. This definition is most consistent with the modern views, but it is not very constructive for measuring purposes.

It is likely (but not guaranteed) that all these masses: inertial, gravitational, and energybased, describe the same physical quantity that we label as $m$ in our equations. In this laboratory we will measure mass of an object using the gravitational force between the Earth and the object. The S.I. units for masses are kilograms (kg). To convert from kg to $g$, we merely multiply by a factor of 1000 .

Volume is simply the amount of space that an object occupies or takes up. There is a myriad of units for volume, which include liters ( $l$ ), cubic centimeters $\left(\mathrm{cm}^{3}\right)$, and cubic meters $\left(\mathrm{m}^{3}\right)$. One can of course convert from any one of these units into the other by knowing the proper conversion factor, e.g., there are $1000 \mathrm{~cm}^{3}$ in 1 l . In this laboratory we will compute the volumes and the densities of various metal cylinders. In the process, we will have to determine the (cross-sectional) diameter $D$ of each cylinder or its radius $R$. The volume $V$ of a cylinder is the cross-sectional area $A$ (or area of any of the circular faces) multiplied by the height $H$ of the cylinder such that $V=A H$. From elementary geometry, the cross-sectional area is given by $A=\pi R^{2}=\pi D^{2} / 4$. Therefore, we can write the volume of a cylinder as

$$
\begin{equation*}
V=\pi R^{2} H=\frac{1}{4} \pi D^{2} H \tag{1}
\end{equation*}
$$

Density. We can use the notions of mass and volume to characterize an object. In fact, the ratio of mass to volume for a given object is what we call its density, and this property is specific to a kind of material. For instance, the density of copper is greater than that of zinc. Furthermore, if we consider two spheres identical in size (and therefore, in volume) yet different in the type of material making them up, such that one is made of copper and the other, of zinc, the implication here is that the mass of the copper sphere has to be larger (since the density of it is larger). We can understand this better through the mathematical definition of density, namely,

$$
\begin{equation*}
\rho=\frac{m}{V} \tag{2}
\end{equation*}
$$

where $m$ represents the mass of the object and $V$ is the volume of the object. The density of an object can be specified in units of grams per cubic centimeters ( $\mathrm{g} / \mathrm{cm}^{3}$ ) or kilograms per cubic meters $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$. From equation (2), we see that the density of an object is directly proportional to its mass, such that if the mass is larger, the density is greater (assuming the volume is kept the same), as had been the case in the example of the copper and zinc spheres previously mentioned. The density of an object is also inversely proportional to the volume, such that if the volume decreases, the density increases (keeping the mass the same), as is the case if comparing a 1 kg wooden block with a 1 kg solid steel block (the steel block is of a smaller volume). In short, density is a measure of the compactness of an object. Note that if we consider two copper spheres, one larger in size than the other, the one will have more mass than the other since the density of copper is a fixed value.

Uncertainty of Measurements (also known as the error): When experimentalists measure some quantity from their data, they generally assume that some exact or "true value" exists. Experimentalists reporting their results usually specify a range of values that they expect this "true value" to fall within. The most common way to show the range of values is:

$$
\text { measurement }=\text { best estimate } \pm \text { uncertainty. }
$$

For example, consider the following measurement of length: $10 \mathrm{~cm} \pm 0.1 \mathrm{~cm}$. This measurement implies that the experimenter is confident that the actual value for the length being measured lies somewhere between 9.9 cm and 10.1 cm . The uncertainty is the experimenter's best estimate of how far an experimental quantity might be from the "true value."

Combining Measurement: Consider the formula for density $\rho=\frac{m}{V}$, where $m$ and $V$ have uncertainties of $\delta m$ and $\delta V$, respectively. What is the correct way to define uncertainty of density, that is, $\delta \rho$ ? Adding the absolute values of the uncertainties, we overestimate the total uncertainty since the uncertainties can be either positive or negative. The correct statistical rule is to add the fractional uncertainties in quadrature, i.e.

$$
\begin{equation*}
\frac{\delta \rho}{\rho}=\sqrt{\left(\frac{\delta V}{V}\right)^{2}+\left(\frac{\delta m}{m}\right)^{2}} \tag{3}
\end{equation*}
$$

where

$$
\begin{equation*}
\frac{\delta V}{V}=\sqrt{\left(\frac{\delta H}{H}\right)^{2}+\left(\frac{2 \delta D}{D}\right)^{2}}=\sqrt{\left(\frac{\delta H}{H}\right)^{2}+\left(\frac{2 \delta R}{R}\right)^{2}} . \tag{4}
\end{equation*}
$$

The uncertainties of a measurement depend on the intricate nature of the experiment, and how best the experimenter feels assured that he/she is taking a precise measurement. In our case, the uncertainties in mass and length measurements can be regarded as the least count (smallest unit of division) of our measuring instruments:

Mass measured with Triple-Beam Balance: $\delta m= \pm 0.1 \mathrm{~g}$
Mass measured with Digital Balance: $\delta m= \pm 0.001 \mathrm{~g}$
Length measured with Ruler: $\delta H= \pm 0.5 \mathrm{~mm}$
Length measured with Vernier Caliper: $\delta H= \pm 0.05 \mathrm{~mm}$
Length measured with Micrometer: $\delta H= \pm 0.005 \mathrm{~mm}$
In summary, the uncertainty is the precision to which an experimenter feels he/she can best make an accurate measurement.

## Equipment:

1. Triple-Beam Balance with a set of standard weights.
2. Ruler.
3. Vernier Caliper.
4. Micrometer

## Procedure:

1. Measure the Height $H$ and Diameter $D$ of two weights and calculate the relative errors in these measurements. Use first the ruler, and report your data in Table A. Then, repeat with the use of the Vernier Caliper, record this data in Table B. For the third set of measurements use the micrometer and record your results in Table C

Note 1: your instructor will demonstrate proper use of the Vernier Caliper and the micrometer
Note 2: there are guidelines (see the last page) that were written to help you to manage the error calculations. Please follow the instruction from there and record the corresponding errors in last columns of the following tables.

Table A
With Ruler:

| Sample | Height, (cm) <br> H | Diameter, (cm) $D$ | Error in Height $\frac{\delta H}{H} \times 100 \%$ | Error in Diameter, $\frac{\delta D}{D} \times 100 \%$ |
| :---: | :---: | :---: | :---: | :---: |
| 1. |  |  |  |  |
| 2. |  |  |  |  |

Table B
With Vernier Caliper:

| Sample | Height, (cm) | Diameter, (cm) <br> $H$ | Error in Height, <br>  |  |
| :--- | :---: | :---: | :---: | :---: |
| 1. |  |  | Error in Diameter, <br> $H$ $100 \%$ | $\frac{\delta D}{D} \times 100 \%$ |

$\underline{\text { Table C }}$
With Micrometer:

| Sample | Height, (cm) | Diameter, (cm) | Error in Height, | Error in Diameter, |
| :--- | :---: | :---: | :---: | :---: |
| $H$ | $D$ | $\frac{\delta H}{H} \times 100 \%$ | $\frac{\delta D}{D} \times 100 \%$ |  |

2. Measure the Mass $m$ of each weight using the triple-beam balance and the working analytic balance. Record relevant data in Table D.

Table D-1

| Sample | Triple Beam Balance <br> Mass measurement (g) <br> $m$ | Analytic Balance Mass <br> measurement (g) <br> $m$ | Error in <br> Mass, |
| :--- | :--- | :--- | :--- |
| 1. |  |  | $\frac{\delta m}{m}$ <br> $\times 100 \%$ |
| 2. |  |  |  |

3. Calculate the Volume $V$, the Density $\rho$, and their relative Errors. Incorporate length measurements with the ruler (from Table A) and with the Vernier caliper (from Table B). Record relevant data in Tables D and E correspondingly.

## Table E

$\underline{\text { Ruler + Triple-Beam Balance: }}$

| Sample | Mass, <br> (g) <br> m | Volume, $\left(\mathrm{cm}^{3}\right)$ $V=\frac{1}{4} \pi D^{2} H$ | Density, (g/cm $\left.{ }^{3}\right)$ $\rho=\frac{m}{V}$ | Error in Volume, $\frac{\delta V}{V} \times 100 \%$ | Error in Density, $\frac{\delta \rho}{\rho} \times 100 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. |  |  |  |  |  |
| 2. |  |  |  |  |  |

Table F
Vernier Caliper + Triple-Beam Balance:

| Sample | Mass, <br> (g) <br> m | Volume, $\left(\mathrm{cm}^{3}\right)$ $V=\frac{1}{4} \pi D^{2} H$ | Density, ( $\mathrm{g} / \mathrm{cm}^{3}$ ) $\rho=\frac{m}{V}$ | Error in Volume, $\frac{\delta V}{V} \times 100 \%$ | Error in Density, $\frac{\delta \rho}{\rho} \times 100 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. |  |  |  |  |  |
| 2. |  |  |  |  |  |

## Table G

$\underline{\text { Micrometer + Triple-Beam Balance: }}$

| Sample | Mass, <br> (g) <br> m | Volume, $\left(\mathrm{cm}^{3}\right)$ $V=\frac{1}{4} \pi D^{2} H$ | Density, ( $\mathrm{g} / \mathrm{cm}^{3}$ ) $\rho=\frac{m}{V}$ | Error in Volume, $\frac{\delta V}{V} \times 100 \%$ | Error in Density, $\frac{\delta \rho}{\rho} \times 100 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. |  |  |  |  |  |


| 2. |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |

Table H
Ruler + Analytic Balance:

| Sample | Mass, <br> $(\mathrm{g})$ | Volume, $\left(\mathrm{cm}^{3}\right)$ <br> $m$ | Density, $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ <br> $\mathrm{V}=\frac{1}{4} \pi D^{2} H$ | Error in <br> Volume, | Error in <br> Density, <br> m |
| :--- | :--- | :---: | :---: | :--- | :--- |
|  |  |  | $\frac{\delta V}{V} \times 100 \%$ | $\frac{\delta \rho}{\rho} \times 100 \%$ |  |
| 1. |  |  |  |  |  |
| 2. |  |  |  |  |  |

## Table I

Vernier Caliper + Analytic Balance:

| Sample | Mass, <br> $(\mathrm{g})$ | Volume, $\left(\mathrm{cm}^{3}\right)$ | Density, $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ | Error in <br> Volume, | Error in <br> Density, |
| :--- | :--- | :---: | :---: | :--- | :--- |
|  | $m$ | $V=\frac{1}{4} \pi D^{2} H$ | $\rho=\frac{m}{V}$ | $\delta V$ <br> $V$ |  |
| 1. |  |  |  | $\frac{\delta \rho}{\rho} \times 100 \%$ |  |

Table J
Micrometer + Analytic Balance Balance:

| Sample | Mass, <br> $(\mathrm{g})$ | Volume, $\left(\mathrm{cm}^{3}\right)$ | Density, $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ | Error in <br> Volume, | Error in <br> Density, |
| :---: | :--- | :---: | :---: | :--- | :--- |
|  | $m$ | $V=\frac{1}{4} \pi D^{2} H$ | $\rho=\frac{m}{V}$ | $\frac{\delta V}{V} \times 100 \%$ | $\frac{\delta \rho}{\rho} \times 100 \%$ |


| 1. |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 2. |  |  |  |  |  |

4. Compare your density calculations with a table of known densities shown in Table F. Fill according entries in Table G, and deduce material type.

Table K: Densities of Common Substances

| Material | Density, $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ | Material | Density, $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ |
| :--- | :--- | :--- | :--- |
| Aluminum | 2.7 | Brass | 8.6 |
| Zinc | 7.1 | Copper | 9.0 |
| Iron | 7.9 | Lead | 11.4 |

## Table L: Results

| Sample | Most accurate density calculation <br> within the range, $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ <br> $\rho \pm \delta \rho$ | Closest density <br> value from the <br> Table $\mathrm{F},\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ | Conclusion about <br> the material type |
| :--- | :--- | :--- | :--- |
| 1. |  |  |  |
| 2. |  |  |  |

## Discussion:

## Questions (to be answered and included in your Discussion section):

a. Determine which measuring device contributed more to any error and explain why. What would you suggest to improve the accuracy of the measurement?
b. Why does it make more sense to provide relative errors, such as: $\frac{\delta V}{V}, \frac{\delta m}{m}, \frac{\delta \rho}{\rho}$, instead of absolute errors: $\delta V, \delta m, \delta \rho$ ? Before you answer, think of a measurement of your height with an error of 10 cm and a measurement of the distance from New York to Los Angeles with the same error. What can you say about the absolute and relative errors for these measurements? Which one is more accurate?
c. Consider the errors involved in the measurements you made in this lab in both length and mass. Search the internet to find devices that exceed the precision of devices used in this lab. Can you find the maximum experimental precision of mass and length measurement that has currently been achieved?

## Free Fall: Picket Fence Pre-lab

1) What is the goal of this experiment? Summarize in a few sentences.
2) How will you experimentally determine the acceleration due to gravity using the LabQuest?
3) Does it matter how you drop the picket fence? Why or why not?
4) How do the LabQuest and photogate produce the graphs we will use? In other words, how does the equipment work?

## Free Fall: Picket Fence

## Objectives:

The goals of this experiment are to visually understand the acceleration due to gravity near the Earth's surface $g$, and to experimentally measure the value of $g$.

## Introduction:

An object is in free fall if the only force acting on it is gravity. The acceleration due to gravity, $g$, can be considered constant for objects moving close enough to the Earth's surface.

It is an interesting result that $g$ is independent of the mass of the object. However, the value of $g$ varies with the distance from Earth and depends on the mass of the Earth and the distance between the object and the center of the Earth. If you are on Earth's surface, then:

$$
\begin{equation*}
g=G \frac{M_{E}}{R_{E}^{2}}, \tag{1}
\end{equation*}
$$

where $M_{E}$ and $R_{E}$ are the mass and radius of the Earth and $G$ is and the universal gravitational constant. The experimentally determined value of the gravitational acceleration near the Earth's surface is $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$. The value of $g$ was first determined experimentally by Galileo, who measured the time it took for various objects to roll down an inclined plane.

Acceleration is defined as the rate at which an object changes its velocity. Any constant acceleration motion, including free fall, is expressed by a simple equation:

$$
\begin{equation*}
v_{f}=v_{i}+a \Delta t \tag{2}
\end{equation*}
$$

where $v_{i}$ and $v_{f}$ are the initial and the final velocities, and $a$ is the constant acceleration, and $\Delta t$ is the time elapsed from when the object had its initial velocity $v_{i}$ up to when it obtains its final velocity, $v_{f}: \Delta t=t_{f}-t_{i}$. Equation (2) corresponds to a standard linear dependence (velocity $v s$. time) of the general form $y=m x+b$, where $m$ is the slope and $b$ is the $y$-intercept. In the specific case of Eq.(2), the initial velocity $v_{i}$ is the $y$-intercept and the acceleration $a$ is the slope. Eq.(2) can be rearranged to solve for acceleration:

$$
\begin{equation*}
a=\frac{v_{f}-v_{i}}{\Delta t} \tag{3}
\end{equation*}
$$

Imagine the motion of an object in free fall dropped straight down, say a stone dropped from a bridge. To be specific, imagine that you could take very fast exposure photos of the stone as it falls, at a specific frequency (say 100 photos every second, such that 1 photo is recorded every 0.01 second). What would the sequence of photos look like? Think carefully about what acceleration means. If an object falls with a constant acceleration of $10 \mathrm{~m} / \mathrm{s}^{2}$, this means that every second the speed increases by $10 \mathrm{~m} / \mathrm{s}$. After 1 second, the object is moving at $10 \mathrm{~m} / \mathrm{s}$.

After 2 seconds, it is moving at $20 \mathrm{~m} / \mathrm{s}$ - and so it has covered more distance between 1-2 seconds than it did between $0-1$ seconds. Therefore, the sequence of photos will show an increase in the separation between successive frames.

Since the acceleration is constant, the average and instantaneous accelerations are equivalent. The velocity however, is changing at a constant rate. We can calculate the instantaneous velocity $v_{n}$ at some time $t_{n}$ by instead computing the average velocity over a time interval where that time occurs in between the two endpoints:

$$
\begin{equation*}
v_{n}=\frac{\Delta x_{n+1, n-1}}{\Delta t_{n+1, n-1}}=\frac{x_{n+1}-x_{n-1}}{t_{n+1}-t_{n-1}} . \tag{4}
\end{equation*}
$$

For example, the instantaneous velocity at time $t_{3}$ is $v_{3}=\frac{\Delta x_{4,2}}{\Delta t_{4,2}}$, where $\Delta x_{4,2}$ is the distance between points 4 and 2 .

## Equipment:

1. Photogate.
2. Lab Quest 2
3. Picket Fence.
4. Soft surface for Picket Fence to land on (Backpacks are fine)


## Procedure:

Fasten the Photogate rigidly to a ring stand so the arms extend horizontally, as shown in Figure 1. The entire length of the Picket Fence must be able to fall freely through the Photogate. To avoid damaging the Picket Fence, make sure it has a soft surface (such as a carpet) to land on.

Set up shown Below:


Connect the Photogate to the DIG 1 input of the LabQuest2.
The photogate will not automatically load in the Lab Quest2 and must be manually set up. In order to do this, select the sensor icon in the top left of the screen. You will see a blank menu. On this menu select "Sensors" located at the top of the Screen and select Sensor Setup.

Select the bar that reads "No Sensor" next to DIG 1. A list of sensors will appear. Select photogate from the list

Once the photogate is selected and you return to the sensor menu you will see a box that says unblocked in big letters.

Block the Photogate with your hand; note that the Photogate is shown as blocked. Remove your hand and the display should change to unblocked.

Make sure that the photogate is correctly calibrated for your picket fence in the sensors menu:

On the sensor menu click Mode, located on the right of the screen. It should read "time based"

On the bottom of this menu expand the option "Photogate Mode: Motion"
The sensor default setting should read "Vernier Picket Fence" which is what you are using.
Select the "graph" Button in the top right-hand corner of the LabQuest. After the graphs are on the screen press the green "play" button in the lower right-hand corner to activate the Photogate.

Hold the top of the Picket Fence and drop it through the Photogate, releasing it from your grasp completely before it enters the Photogate.

Be careful when releasing the Picket Fence. It must not touch the sides of the Photogate as it falls, and it needs to remain vertical. Click the stop button to end data collection. Note: When you push the play button it turns into the stop button.

Examine your graphs. The slope of a velocity $v s$. time graph is a measure of acceleration. If the velocity graph is approximately a straight line of constant slope, the acceleration is constant.

If the acceleration of your Picket Fence appears constant, fit a straight line to your data. To do this, click on the velocity graph once to select it, then click to fit the line $y=m x+b$ to the data, which can be found under Analyze- Curve Fit- Linear. Record the slope in the data table.

To establish the reliability of your slope measurement, repeat this process five more times. Do not use drops in which the Picket Fence hits or misses the Photogate. Record the slope values in the data table.

## DATA TABLE:

| Trial | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| Slope $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |  |  |  |  |  |  |


|  | Minimum | Maximum | Average |
| :--- | :--- | :--- | ---: |
| Acceleration $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |  |  |  | | Acceleration due to gravity, $g$ |  | $\mathrm{~m} / \mathrm{s}^{2}$ |
| :--- | ---: | ---: |
| Precision |  | $\%$ |

Draw one of your Velocity Vs Time Graphs Below and place it in your lab report or take a picture of your LabQuest readouts with your phone and include it in your lab report

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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## Discussion:

1. Explain why your velocity vs time graph should be a straight line.
2. Why should the slope your velocity vs time graph be equal to the acceleration due to gravity, $g$ ?
3. Compute the uncertainty in your measurement using the following equation: $\delta g=g \sqrt{\left(\frac{4 \delta t}{t}\right)^{2}+\left(\frac{\delta x}{x}\right)^{2}}$ and report your result as $\boldsymbol{g} \pm \boldsymbol{\delta} \boldsymbol{g}$. Refer back to "Measurement and Density" laboratory, if needed, for a discussion on computing the uncertainty in a measurement. You will have to explore LabQuest to determine how these quantities are handled by the program.
4. We do not see feathers and baseballs fall at the same rate on Earth's surface. Why?
5. Discuss the possible sources of error in your measurement. Specifically, what are some sources of systematic error?
6. Why don't we calculate the instantaneous velocity as

$$
v_{n}=\frac{\Delta x_{n, n-1}}{\Delta t_{n, n-1}} \text { or } v_{n}=\frac{\Delta x_{n+1, n}}{\Delta t_{n+1, n}},
$$

why do we use Eq.(4) instead?
7. The value of $g$ you obtain in this experiment is accurate at sea level however as you increase in altitude the value of $g$ decreases the change in $g, \Delta g$ or the change in $g$ for a given altitude is given by the following equation

$$
\Delta g=g_{a l t}-g=G M_{E}\left(\frac{1}{(R+h)^{2}}-\frac{1}{(R)^{2}}\right)
$$

Where G is the universal gravitation constant, $M_{E}$ is the Mass of the earth, R is the average radius of the earth and $h$ is the altitude above sea level. We will calculate $\Delta g$ to determine the required resolution to measure differences in altitude from large altitudes to small altitudes, use the equation above to complete the table of given altitudes.

| $\Delta g$ | h |
| :---: | :---: |
|  | 1 cm |
|  | 1 m |
|  | 1 km |
|  | 100 km |
|  | 1000 km |

Research an instrument called a gravity gradiometer and try to determine the current limits on measuring $\Delta g$ also interpret the quantity $\Delta g$ for large values of h and when h is identically zero.

## 2D Projectile Motion: Pre-lab

1) What is the goal of the experiment? Summarize in a few sentences.
2) How many times do you need to measure the height of the table?
3) Why do we need two photogates?
4) How will we use the photogates and the LabQuest to determine the initial velocity?

## 2D Projectile Motion

## Objectives:

The goal of this lab is to visually and conceptually understand the properties of projectile motion. You will experimentally measure the horizontal distance traveled by a small metal ball undergoing 2D motion and compare your value with the theoretical value.

## Introduction:

An object moving in 2D can be regarded as exhibiting two independent motions along the x and y axes. The basic equation for the $x$ and $y$ displacements as a function of time for an object moving with a constant acceleration and neglecting the effects of air resistance are:

$$
\begin{align*}
& \Delta x=v_{0 x} \Delta t+\frac{1}{2} a_{x}(\Delta t)^{2}  \tag{1}\\
& \Delta y=v_{0 y} \Delta t+\frac{1}{2} a_{y}(\Delta t)^{2} \tag{2}
\end{align*}
$$

Without air resistance, the $x$-component of the initial velocity remains constant throughout the trajectory, and therefore, $a_{x}=0$. Furthermore, $a_{y}$ is a non-zero constant and is equal to $-g$. These two facts allow us to rewrite equations (1) and (2) as:

$$
\begin{align*}
& \Delta x=v_{0 x} \Delta t  \tag{3}\\
& \Delta y=v_{0 y} \Delta t-\frac{1}{2} g(\Delta t)^{2} \tag{4}
\end{align*}
$$

If the initial velocity only contained an $x$ component (this is equivalent to saying that the launch angle $\theta=0^{\circ}$ ), such as a projectile fired purely horizontally, then equation (4) reduces to simply the equation for free-fall motion:

$$
\begin{equation*}
\Delta y=-\frac{1}{2} g(\Delta t)^{2} \tag{5}
\end{equation*}
$$

We can take the downward direction to be positive such that equation (5) may now be rewritten as:

$$
\begin{equation*}
h=\frac{1}{2} g(\Delta t)^{2} \tag{6}
\end{equation*}
$$

where $h$ is the height above the ground to the point at which the projectile had been fired. We are interested in predicting the value of the horizontal distance travelled by an object in such a motion. The predicted value $\Delta x_{\text {pred }}$ can be found by combining equations (3) and (6):

$$
\begin{equation*}
\Delta x_{p r e d}=v_{0 x} \sqrt{\frac{2 h}{g}} \tag{7}
\end{equation*}
$$

The experimental value $\Delta x_{\text {exp }}$ can be measured directly with a ruler.

## Equipment:

1. Tracks
2. Small metal ball (pinball)
3. Meter stick
4. Cardboard (or paper)
5. Tape
6. Ring stand and C-clamps
7. Two Vernier photogates


## Procedure:

Data Collection: To find the predicted value of $\Delta x$, you need to know $v_{0 x}$, which is the initial velocity that the ball has as it rolls off the table. We will let the ball roll down a ramp, at which point it will be accelerating. Upon leaving the ramp, the ball will hit the table before rolling off with nearly constant $v_{0 x}$. If you know the time that the ball takes to travel a known distance, $\Delta t, v_{0 x}$ can be computed using equation (3). After the ball leaves the table, it will land at some location, and from there, you can measure $\Delta x_{\text {exp }}$ directly

1. Use the track and some books to set up a small ramp at your worktable. Use some cardboard or a notebook to smooth out the gap between the ramp and the table for the rolling ball. This will help ensure that the ball rolls off the table with all of the initial velocity in the $x$-direction.

2. Pick two points past the end of the ramp on the table and set up the two photogates between them, using the C-clamps to secure them in place. Both photogates must be past the edge of the ramp. Roll the ball down the track a few times to familiarize yourself with the output on the handheld Vernier device.
3. Before you begin taking measurements, release the ball along the track and locate the approximate landing spot of the ball. Place some cardboard around the landing spot and fix it using some tape. This will facilitate your experimental measurements of $\Delta x$.
4. Using the meter stick, measure the height from the point where the ball rolls off the table to the floor. Record the height in column 2 of Table 1. Think: do you need to do this more than once?
5. Measure the distance between the points you selected in step 2 and record the value in column 3 of Table 1 (distance between photogates). If you don't move the photogates, you'll only need this measurement once.
6. Connect the photogates to DIG 1 and DIG 2 of the LabQuest. If they don't appear on the screen automatically, connect them using the Sensors $\rightarrow$ Photogate option.
7. Release the ball from the top of the track and measure the time it takes the ball to go through the photogates. Record this time in column 4 of Table 1, $\Delta t$.
8. Mark the landing spot of the ball on the cardboard you put on floor. The horizontal distance from the edge of the table to where the ball lands is your $\Delta x_{\text {exp }}$.
9. Repeat steps $7-8$ for 3 trials in total.

Calculations:
Using your values for $\Delta t$ and $v_{0 x}$, calculate $\Delta x_{\text {pred }}$ for each trial and record in column 6 of Table 1. Find the average value of $\Delta x_{\text {pred }}$ and $\Delta x_{\text {exp }}$. Show your work in the space provided below.

Table 1

|  | Height (m) | Distance between <br> photogates (m) | $\Delta t(\mathbf{s})$ | $v_{0 x}(\mathbf{m} / \mathbf{s})=$ <br> Distance <br> between <br> photogates/ <br> $\Delta t$ | $\Delta x_{\text {pred }}(\mathbf{m})$ | $\Delta x_{e x p}(\mathbf{m})$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Trial 1 |  |  |  |  |  |  |
| Trial 2 |  |  |  |  |  |  |
| Trial 3 |  |  |  |  |  |  |
| Averages |  |  |  |  |  |  |

## Discussion:

1. How do your experimental and predicted values of $\Delta x$ compare to each other?
2. The main source of error for the experimental value will be $\delta x$, which is just the accuracy of the meter stick. What are some other sources of error, for both the experimental and predicted values?
3. If you raised the ramp much higher, how would $\Delta x$ change? (Think about how the initial speed of the ball, as it leaves the table, would change.)
4. Describe the acceleration of the ball once it leaves the table and undergoes projectile motion.

# Addition of Two Vectors, 2-D Force Table: Pre-lab 

1) What is the goal of this experiment? Summarize in a few sentences.
2) Does it matter if you convert from grams to kilograms? Why or why not?
3) Why is this lab called "addition of vectors" if it is talking about hanging various weights off a table?
4) What is the difference between the "experimental" $m_{n e t}$ and the "theoretical" $m_{n e t}$ ?

# Lab \#4 Addition of Two Vectors, 2-D Force Table 

by Dr. Xin Gao

## Objective:

The goal of this experiment is to learn how vectors add both analytically and graphically with a 2-D force table. By balancing the force table, students will practice how to combine two vector components to get the magnitude and direction of the net vector.

## Theory:

We know from lecture that given the components of a vector, to find the magnitude and direction (angle) of the vector, we do the following calculations:

$$
\begin{aligned}
& \mathrm{F}_{\text {net }}=\sqrt{ } \mathrm{F}_{\mathrm{x}}{ }^{2}+\mathrm{F}_{\mathrm{y}}{ }^{2} \\
& \Theta=\tan ^{-1}\left(\mathrm{~F}_{\mathrm{y}} / \mathrm{F}_{\mathrm{x}}\right)
\end{aligned}
$$

Where F represents a vector.
In this experiment, we are going to use a 2-D Force Table to learn how to balance 3 forces, which are vectors in 3 different directions, such that the resultant vector is zero. We are going to hang masses on the force table in 3 different positions. The masses will exert forces on the ropes they are connected to due to their weight.

Since mass and weight (which is a force) are linearly related, $\mathrm{F}=\mathrm{mg}$, we will introduce a fictitious vector that we will call " $m$ ". Note: mass in NOT a vector. We are just using this as a label so we don't have to multiply everything by g . This vector m has a magnitude and direction given by:

$$
\begin{align*}
& \mathrm{m}_{\mathrm{net}}=\sqrt{ } \mathrm{m}_{\mathrm{x}}{ }^{2}+\mathrm{m}_{\mathrm{y}}{ }^{2}  \tag{1}\\
& \Theta=\tan ^{-1}\left(\mathrm{~m}_{\mathrm{y}} / \mathrm{m}_{\mathrm{x}}\right) \tag{2}
\end{align*}
$$

which help us directly figure out how much mass we need to put on each hanger to make the force table reach equilibrium. However, if we use the above to find the resultant from two of the masses, to balance them out, we need to add $180^{\circ}$ after we find the angle to find where the $3^{\text {rd }}$ mass must be hung to balance the other two.

For convenience, we choose the direction along $0^{\circ}$ as our +x direction and the direction along $90^{\circ}$ as our $+y$ direction. So if you are given a vector in any direction, you should decompose it into x and y directions to get the $\mathrm{x}, \mathrm{y}$ components, then combine them together to get the magnitude and angle of the combined vector.


The ways to tell if the force table achieves balance is just to simply look at the ring in the middle of the force table and see if it's exactly in the middle, not touching any part of the little pole at the center.


## Procedure:

1. Set up the force table with three hangers attached to a circular ring with inelastic strings.
2. Place the ring in the center around the pole of the force table.
3. Place the first hanger along the $0^{\circ}$ direction.
4. Place the second hanger along $120^{\circ}$ direction.
5. Add 100 grams on top of the first and the second hanger for the first trial. Remember to add the mass of the hanger itself, 50 grams for future calculation of the results.
6. The mass on the first hanger along $0^{\circ}$ direction will be $\mathrm{m}_{\mathrm{x}}$ since it is along the x axis.
7. Figure out the x and y components of the force (actually the mass) in the string attaching the second hanger and express them as $\mathrm{m}_{2 \mathrm{x}}$ and $\mathrm{m}_{2 \mathrm{y}}$.
8. We want to find the resultant vector that you get from adding $m_{1}$ and $m_{2}$. The first mass only has an x -component and you found the x and y components of the second mass in Step 7. Use the formula $\mathrm{m}_{\text {net }}=\sqrt{ } \mathrm{m}_{\mathrm{x}}{ }^{2}+\mathrm{m}_{\mathrm{y}}{ }^{2}$ to find the theoretical value of the total mass. This will also be how much mass you need to add to the $3^{\text {rd }}$ position to balance the other two.
9. Use the formula $\Theta=\tan ^{-1}\left(m_{y} / m_{x}\right)$ to also find the angle of the resultant $\left(m_{\text {net }}\right)$.
10. Add $180^{\circ}$ to $\Theta$ in order to find the opposite direction of the resultant, where the third hanger should be placed to make the force table in equilibrium.
11. Now you know the theoretical value of $\mathrm{m}_{\text {net }}$, using it as a guide and find out what mass you need to put on the third hanger by trial and error, which means you need to manually balance the force table by adding masses on the hanger in the third direction. This value you get from experiment will be your experimental value of $\mathrm{m}_{\text {net. }}$. Don't forget that the hanger itself is $\mathbf{5 0 g}$ !
12. Double check the circular ring to see if it stays in the center, not touching any part of the pole to make sure the whole system is at balance.
13. Record the angle and total mass on the third hanger (the mass you put on the hanger plus the hanger itself), record it and compare its magnitude to the theoretical value you calculated using equation (1) and (2).
14. Put 100 grams mass on the hanger along the $0^{\circ}$ direction and 150 grams mass on the hanger along the $120^{\circ}$ direction and repeat step 8-12.

## Data \& Calculation

## Trial 1

|  | Hanger \#1 <br> $\left(\mathrm{m}_{\mathrm{x}}\right)$ | Hanger \#2 <br> $\left(\mathrm{m}_{\mathrm{y}}\right)$ | Hanger \#3 <br> $\left(\mathrm{m}_{\text {net }}\right)$ <br> Theoretical | Hanger <br> $\# 3\left(\mathrm{~m}_{\text {net }}\right)$ <br> Experimental | Error\% |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Angle ( ${ }^{\circ}$ ) |  |  |  |  |  |
| Mass (g) |  |  |  |  |  |

## Show your calculations below:

## Trial 2

|  | Hanger \#1 <br> $\left(\mathrm{m}_{\mathrm{x}}\right)$ | Hanger \#2 <br> $\left(\mathrm{m}_{\mathrm{y}}\right)$ | Hanger \#3 <br> $\left(\mathrm{m}_{\text {net }}\right)$ <br> Theoretical | Hanger <br> $\# 3\left(\mathrm{~m}_{\text {net }}\right)$ <br> Experimental | Error\% |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Angle ( ${ }^{\circ}$ ) |  |  |  |  |  |
| Mass (g) |  |  |  |  |  |

Show your calculations below:

## Questions:

1. Do your experimental values match the theoretical values from calculation? If not, explain the possible reasons or sources of error.
2. Why do we need to add 180 degrees to the calculated angle to find the $3^{\text {rd }}$ direction?

## Atwood's Machine: Pre-lab

1) What is the goal of this experiment? Summarize in a few sentences.
2) This experiment has two parts. Will any of the columns for either part contain the same value? Why or why not?
3) How will we use the LabQuest and the photogate to experimentally determine the acceleration of the system?
4) How will you use your data to experimentally determine the acceleration due to gravity for each part?

## Atwood's Machine

## Objectives:

The goals of this lab are to: (1) test Newton's Laws of Motion, (2) determine the relationship between the masses on an Atwood's machine and the acceleration, (3) measure acceleration due to gravity.

## Introduction:

Atwood's machine is a simple elevator: two weights are hung over a pulley and the system will begin to accelerate due to gravity. The weight of the descending mass is partly counterbalanced by the weight of the ascending mass, so that the acceleration of the system is a fraction of gravity acceleration $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$.

According to Newton's Second Law the vector sum of all forces acting on any system is equal to the mass of the system times its acceleration:


Figure 1

$$
\begin{equation*}
\vec{F}_{N e t}=m \vec{a} . \tag{1}
\end{equation*}
$$

Based on the free body diagram, see Figure 2, and assuming that the pulley is massless and frictionless, and the string has no mass and doesn't stretch we can write

$$
\begin{gather*}
T-m_{2} g=m_{2} a,  \tag{2}\\
T-m_{1} g=-m_{1} a,
\end{gather*}
$$


where $T$ is the tension in the string, $a$ is the acceleration of the
Figure 2 system, $g$ is the acceleration due to gravity, and $m_{1}>m_{2}$.
Solving Eq. 2 for the system acceleration $a$ yields:

$$
\begin{equation*}
a=\frac{m_{1} g-m_{2} g}{m_{1}+m_{2}} . \tag{3}
\end{equation*}
$$

It is interesting to mention that if you treat the entire system of two masses and string as one object that accelerates together (which it does), then the force accelerating the system down is $m_{1} g$, and a force of $m_{2} g$ resists this acceleration by pulling the opposite direction of the actual motion. So, the net force acting on the system is $F_{N e t}=\left(m_{l}-m_{2}\right) g$, while the total mass of the system is $\left(m_{l}+m_{2}\right)$. Applying Newton's second law (1) for the entire system gives us the same results as in (3):

$$
\begin{equation*}
a=\frac{F_{N e t}}{m}=\frac{\left(m_{1}-m_{2}\right) g}{m_{1}+m_{2}}=g \frac{m_{1}-m_{2}}{m_{1}+m_{2}}=g \frac{\Delta m}{m} . \tag{4}
\end{equation*}
$$

Note that torque required to cause the pulley to begin spinning is assumed to be negligible, as is the mass, friction, and rotational inertia of the pulley.

## Equipment:

1. LabQuest data-collection device
2. Vernier Photogate with Super Pulley
3. Mass set
4. String

## Procedure:

1. Part I: Constant Total Mass. For this part of the experiment you will keep the total mass used constant, but move weights from one side to the other, so that the difference in masses changes.
2. Set up the Atwood's machine apparatus. Be sure the heavier mass can move at least 40 cm before striking the floor.
3. Connect the Photogate with Super Pulley to DIG1 of the interface.
4. Set the sensor set-up for DIG1 to "photogate" mode. Under the "data collection" menu, set to "pulley" and "inner groove."
5. Arrange a collection of masses totaling 200 g on $m_{2}$ and a 200 g mass on $m_{l}$. What is the acceleration of this combination? Record your values for mass and acceleration in
the data table.
6. Move 5 g from $m_{2}$ to $m_{1}$. Record the new masses in the data table.
7. Position $m_{l}$ as high up as it can go. Click the green start arrow to begin data collection and hit the red stop box to stop (time this so one student catches the masses before they hit the floor and another student hits stop). Steady the masses so they are not swinging. Wait one second and release the masses. Catch the falling mass before it strikes the floor or the other mass strikes the pulley.
8. Click the "Analyze" button and select the region of the velocity-versus-time graph where the velocity was increasing at a steady rate. Click the "Curve Fit" button and "Velocity." Select "Linear Fit" to fit the line $y=m x+b$ to the data. Record the slope, which is the acceleration, in the data Table 1.
9. Continue to move masses from $m_{2}$ to $m_{1}$ in 5 g increments, changing the difference between the masses, but keeping the total constant. Repeat Steps 6-7 for each mass combination.
10. Part II: Constant Mass Difference. For this part of the experiment you will keep the difference in mass between the two sides of the Atwood's machine constant and increase the total mass.
11. Put 120 g on $m_{l}$ and 100 g on $m_{2}$.
12. Repeat Steps 6-7 (the previous part) to collect data and determine the acceleration.
13. Add mass in 20 g increments to both sides, keeping a constant difference of 20 grams. Record the resulting mass for each combination in the data Table 2. Repeat Steps 6-7 for each combination. Repeat the procedure until you get at least five different combinations.

Table 1. The Total Mass Constant

| Trial | Mass, $m_{l}$ <br> (in g) | Mass, $m_{2}$ <br> (in g) | Acceleration, $a$ <br> (in m/s ${ }^{2}$ ) | Mass Difference, <br> $\Delta m=m_{1}-m_{2}$ <br> (ing) | Total Mass, <br> $\mathrm{m}=\mathrm{m}_{1}+m_{2}$ <br> (ing) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |

Table 2. The Mass Difference Constant

| Trial | Mass, <br> $m_{1}$ <br> (ing) | Mass, <br> $m_{2}$ <br> (ing) | Acceleration, $a$ <br> (in m/s ${ }^{2}$ ) | Mass <br> Difference, <br> $\Delta m=m_{1}-m_{2}$ <br> (in g) | Total Mass, <br> $\mathrm{m}=\mathrm{m}_{1}+$ <br> $m_{2}$ <br> (ing) | Inverse <br> Mass, $1 / m$ <br> $\left(\right.$ in g $\left.\mathrm{g}^{-1}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |

3. Analysis
a. For each trial, calculate the difference between $m_{1}$ and $m_{2}$. Enter the result in the column labeled $\Delta m$.
b. For each trial, calculate the total mass in grams. Enter the result in the last column labeled $m$.
c. Plot a graph (use Figure 3 below) of acceleration vs. $\Delta m$, using the Part I data (Table 1). Based on your analysis of the graph, what is the relationship between the mass difference and the acceleration of an Atwood's machine? From the slope of your line of best fit, calculate your experimental acceleration due to gravity $g_{\text {exp }}$ and record the results in Table 3, first column. Hint: look at what you plotted and compare to Eq. 4.


Figure 3
d. Plot a graph (use Figure 4 below) of acceleration vs. $1 / m$, using the Part II data (Table 2). Based on your analysis of the graph, what is the relationship between the mass difference and the acceleration of an Atwood's machine? From the slope of your line of best fit, calculate the acceleration due to gravity $g_{\text {exp }}$ and record the results in Table 3, second column. Hint: look at what you plotted and compare to Eq. 4.


Figure 4
4. Results

Table 3. Results

| Gravity acceleration, <br> $\exp ($ Part I), (in m/s | $\left.\begin{array}{l}\text { Gravity acceleration, } \\ \exp \text { (Part II), (in m/s }\end{array}\right)$ | Gravity acceleration, <br> theory, (in m/s ${ }^{2}$ ) | Error \% <br> (Part I) | Error \% <br> (Part II) |
| :--- | :--- | :--- | :--- | :--- |
|  |  | 9.81 |  |  |

## Discussion:

1. What is a real-world application of Atwood's Machine?
2. Why do the two masses have the same acceleration?
3. Would the acceleration have been different if the Atwood Machine had been started with an initial velocity?
4. For low acceleration values, how would the mass and friction of the pulley affect the values for $a$ ? Explain your answer.
5. For high values of acceleration, the string may actually slip on the pulley wheel. How would this error affect the observed values for the acceleration of the system?

## Static and Kinetic Friction: Pre-lab

1) What is the goal of this experiment? Summarize in a few sentences.
2) This experiment has two data tables that you can fill out from doing the same trial. Why?
3) Draw or describe what the force-time graph should look like.
4) How will you use the force-time graph to determine $\mu_{s}, \mu_{k}, f_{s, \max }$ and $f_{k}$ ?

## Static and Kinetic Friction

## Objectives:

The goals of this lab are to: (1) measure and compare static and kinetic friction forces, (2) measure the coefficients of friction; and (3) determine the relationship between the normal and frictional forces.

## Introduction:

Friction is the force resisting the relative motion of surfaces sliding against each other (or trying to move across each other). There are at least two types of friction - kinetic (sliding) and static friction.

When you attempt to push a heavy box resting on the floor, you need to apply a certain amount of force in order for the box to move. While the box is stationary, static friction is pushing against the external force applied; they are equal in magnitude and opposite in direction. The free-body diagram while you are pushing and the box is stationary is:


Figure 1
As you push harder, static friction increases and remains equal to the value of your pushing force. Eventually, if you push hard enough, the box will move; this means that static friction has reached its maximum magnitude. There are two factors which determine the maximum static frictional force which can occur between two surfaces: how "grippy" the surfaces are (friction coefficient), and how hard the two surfaces are being pushed together (normal force):

$$
\begin{align*}
& f_{s} \leq f_{s, \max }  \tag{1}\\
& f_{s, \max }=\mu_{s} n
\end{align*}
$$

where $\mu_{\mathrm{s}}$ is the coefficient of static friction and depends on the two surfaces that are in contact and $n$ is the normal force. In this particular case where the object is moved on a horizontal surface, the normal force is equal to the weight:

$$
\begin{equation*}
n=m g \tag{2}
\end{equation*}
$$

Once the box moves, it will be opposed by kinetic friction. The equation for kinetic friction looks very similar to that for static friction (1):

$$
\begin{equation*}
f_{k}=\mu_{k} n \tag{3}
\end{equation*}
$$

Now the coefficient of friction is $\mu_{\mathrm{k}}$, which lower than $\mu_{\mathrm{s}}$ for the same two surfaces. This makes sense: it is easier to keep the box moving once you have overcome static friction.

## Equipment:

1. Dual-Range Force Sensor
2. Wooden block
3. Masses
4. String

## Procedure:

In this experiment, you will use a Dual-Range Force Sensor to study static friction and kinetic friction on a wooden block. A Motion Detector will be used to analyze the kinetic friction acting on a sliding block. The set-up is shown in Figure $2^{1}$.


Figure 2

1. Data and Calculations
a. Connect the Force Sensor to DIG 1 of the LabQuest data-collection device.
b. It should start up automatically. Make sure that the force sensor appears as "Force Sensor" and the range is 50 N . The graph on the screen will show force versus time.
c. For your first trial, measure the total mass of the block with the Force Sensor and record it in column 2 of Tables 1 and 2. Think about how you can do this.
d. Tie one end of a string to the hook on the Force Sensor and the other end to the hook on the block. Place some masses on top of the block. Practice pulling the block and masses horizontally with the Force Sensor along a straight line. Gradually, increase the force until the block starts to slide, and then keep the block moving at a constant

[^0]spee
e. Practice until your force-time graph shows two distinct regions: one part with a positive slope (static friction regime) and the other with nearly zero slope (kinetic friction regime).
f. Examine the force versus time output data from LabQuest. The maximum value of the force is equal to the maximum value of static friction for your set up, which occurs when the block starts to move. The average force for the constant velocity motion region is the magnitude of the kinetic frictional force. Record the values of the maximum static and kinetic friction forces in Tables 1 and 2. You should also record the mass of the block plus any masses you placed on top of it.
g. Change the masses you put on the block and repeat Steps c-f for another five masses; record your data in Tables 1 and 2.
h. Using the data in Table 1, make a plot of the maximum static friction force versus the normal force. Label the axes and give your figure a title.
i. Now using the data in Table 2, make a plot of the kinetic friction force versus the normal force. Label the axes and give your figure a title.

Table 1

| Trial | Mass | Normal <br> force ( $n$ ) | Maximum <br> Static <br> friction <br> force <br> $\left(f_{s, \max )}\right.$ | Static <br> Friction <br> coefficients | Average <br> Static <br> Friction <br> coefficient <br> $\left(\mu_{\mathrm{s}}\right)$ | Error\% in <br> $\boldsymbol{\mu}_{\mathrm{s}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ |  |  |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| $\mathbf{4}$ |  |  |  |  |  |  |
| $\mathbf{5}$ |  |  |  |  |  |  |
| $\mathbf{6}$ |  |  |  |  |  |  |

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Table 2

| Trial | Mass | Normal <br> force ( $n$ ) | Kinetic <br> friction <br> force $\left(f_{\mathrm{k}}\right)$ | Kinetic <br> Friction <br> coefficients | Average <br> Kinetic <br> Friction <br> coefficient <br> $\left(\mu_{\mathrm{k})}\right.$ | Error\% in <br> $\boldsymbol{\mu}_{\mathrm{k}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
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| $\square$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\square$ | , |

Figure 3

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Figure 4
2. Analysis
a. Find the slope of your plots of the friction forces versus the normal force (Figures 3 and 4). Show your work here and give the values.
b. What do the slopes of Figures 3 and 4 represent?
c. Calculate the average friction coefficients (both static and kinetic) as

$$
\begin{equation*}
\bar{\mu}=\frac{1}{N} \sum_{i=1}^{N} \mu_{i} \tag{4}
\end{equation*}
$$

where $N$ is the number of trials. Compare the results with the slopes from the previous steps.
d. Calculate the standard deviation for each friction coefficient as

$$
\begin{equation*}
\delta \mu=\sqrt{\frac{1}{N} \sum_{i=1}^{N}\left(\mu_{i}-\bar{\mu}\right)^{2}} \tag{5}
\end{equation*}
$$

e. Compute the error percentages (last columns of Tables 1 and 2) as: $100 \% \times \delta \mu / \mu$

## Discussion:

1. What causes friction?
2. It intuitively feels like the friction force should be directly proportional to the surface area between materials, yet this is never considered in any practical analysis or application. Why is it so?
3. If polishing surfaces decreases friction, then why does polishing metal surfaces make them fuse together?
4. What do we mean when we say that friction converts kinetic energy into heat?
5. To explain why static friction is greater than kinetic some people say: "it is easier to keep the box moving once you have overcome static friction." It sounds like a wrong or incomplete argument because, regardless of friction, it is always harder to start motion (you need to apply a force) than to continue the motion (no force is needed, its Newton's $1^{\text {st }}$ law). Thus, why do you think static friction is greater than kinetic?

## Conservation of Energy: Pre-lab

1) What is the goal of this experiment? Summarize in a few sentences.
2) Show where Eq. (4) came from.
3) Briefly explain how you will use the LabQuest to determine $v_{i}$ and $v_{f}$.
4) Do you plan to push the cart towards or away from the motion sensor? Why?

## Conservation of Momentum \& Energy

## Part I:

## Objective

In an isolated system, the total energy is conserved. External forces that act on the system can change its total energy. In certain cases, the change in the total energy depends upon the path the object takes while the external force is applied. This is an example of a non-conservative force. The goal of this lab is to measure the work done by friction and use this result to determine the coefficient of kinetic friction of the racetrack in which a cart is pushed. We can use this result to analyze our results in the next part of the experiment, where we study the conservation of momentum.

## Introduction

The conservation of energy is a very powerful statement. It tells us that if we know the total energy of a system at any instant in time, then we also know the total energy of the system at all points in time.

In mathematical form, we can write the conservation of energy as:

$$
\begin{equation*}
\Delta K+\Delta U=W+W_{n c} \tag{1}
\end{equation*}
$$



If the system is isolated, then there is no total change in both the kinetic AND potential energy of the system. Instead, energy can only be transferred BETWEEN kinetic and potential energy (like the roller coaster pictured above).

If the system is non-isolated, then external forces act on it. We say that these external forces can do work. The work done by a constant force is defined as:

$$
\begin{equation*}
W=F \Delta r \cos \theta \tag{2}
\end{equation*}
$$

Where theta is the angle between the external force and the displacement of the object. If the force is conservative, then the work done only depends upon the starting point and the ending point, rather than the path itself. Gravitation is one example- the work done by (or against) the gravitational force only depends upon the vertical displacement, i.e. the change in height.

Friction is an example of a non-conservative force. The amount of work it does depends on the path the object take as friction opposes it. Since the force of kinetic friction is always opposite the motion and perpendicular to the surface, we can write that the work done by friction is:

$$
\begin{equation*}
W_{n c}=-f_{k} \Delta r \tag{3}
\end{equation*}
$$

Note: other types of forces can be non-conservative too.
In the experiment, we will place the cart on a level, horizontal track and push the cart along the track. There will be no changes in its potential energy. After the initial push, no external forces will act. Therefore, the conservation of energy dictates that:

$$
\begin{equation*}
K_{i}=K_{f}+f_{k} \Delta r \tag{4}
\end{equation*}
$$

So, if we know how fast the cart is moving between two different instants and the distance between those points, we can experimentally determine the force of kinetic friction.

## Equipment:

1. LabQuest 2
2. Motion Detector
3. Track
4. Cart Without Plunger

## Procedure:

1) Record the mass of the cart in kilograms (it should be written on the top of the cart).
2) Plug one motion sensor into DIG 1 on the LabQuest. It should boot up automatically.
3) Level the track if it isn't already.
4) Place the motion sensor on the track. The motion sensor works by recording how far away the cart is from it. Therefore, the cart moving away from it is considered "positive" in the graphs.
5) Determine the distance over which you will track the motion. Record that as $\Delta r$.
6) Run the program and push the cart.
7) Highlight a few points at the beginning of the motion. Use these to determine the initial velocity (how do I do that?). Record this as $v_{i}$.
8) Highlight a few points at the end of the motion. Use these to determine the final velocity (how do I do that?). Record this as $v_{f}$.
9) Repeat steps 5-8 for different displacements.

## Data:

m:

| Trial | $\Delta r$ | $v_{i}$ | $v_{f}$ | $K_{i}$ | $K_{f}$ | $K_{i}-K_{f}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |
| 7 |  |  |  |  |  |  |
| 8 |  |  |  |  |  |  |
| 9 |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |

## Analysis:

1) If you haven't already, fill out the last column in the data table. What is the relationship between the displacement and the change in kinetic energy? Why?
2) Plot the change in kinetic energy versus the displacement. What do you predict the graph will look like?
3) Determine the line of best fit of the graph. The slope of this line will give you the force of kinetic friction.
4) Use your result from 3) to determine the coefficient of kinetic friction of the track.

## Conservation of Momentum: Pre-lab

1) What is the goal of this experiment? Summarize in a few sentences.
2) This experiment has 3 parts. Does the type of cart you use for each part matter? Why or why not?
3) You'll need to track the motion of two carts before and after they collide. How do you plan to do this?
4) How will you use the sensors and LabQuest to obtain the velocities of the carts?

## Part II:

## Objectives:

We will consider three types of collisions between carts on tracks. You will observe each collision and hypothesize whether it is elastic or inelastic. You will use the theory outlined below or detailed in your lecture and textbook to prove if the collisions are elastic or inelastic.
Additionally, you will calculate energy losses throughout the experiment to test common assumptions made in problem solving exercises

## Introduction:

The law of conservation of momentum states that if there are no external forces acting on a system, the total momentum of that system must conserved in both magnitude and direction. Let's consider a set of particles (for example, stars) interacting with some internal and external forces. We can state:

- there could be billions of them - a whole star cluster, the particles could collide with each other, explode, break apart - all of those forces are internal; they do not count. The momentum of the whole system would not change if the net external force is zero, $\vec{F}_{\text {ext }}=0$.
- the individual particles will change their momentum all the time, because they experience


Figure 1. NGC206-Star Cloud in Andromeda Galaxy a non-zero net force. We are not saying that the individual particles do not experience the momentum change, only total momentum is conserved.

If we use the symbols $\sum p_{i}$ and $\sum p_{f}$ to denote the total initial and final momentum, respectively, along a particular direction (e.g. the $x$ axis), the momentum conservation law can be expressed as:

$$
\begin{equation*}
\sum_{i} p_{i}=\sum_{f} p_{f} \tag{1}
\end{equation*}
$$

If our system of interest consists of only two objects, the above equation can be explicitly expressed as (since $p=m v$ ):

$$
\begin{equation*}
m_{1} v_{1 i}+m_{2} v_{2 i}=m_{1} v_{1 f}+m_{2} v_{2 f} \tag{2}
\end{equation*}
$$

where $m_{1}$ and $m_{2}$ are the masses of the carts, respectively, and $v_{\alpha i(f)}$ is the velocity of the $\alpha^{\text {th }}$ cart just before (just after) the collision. Note: this is a vector equation. You must account for magnitude and direction.

In the case of an elastic collision, the total kinetic energy is also conserved, and we therefore have the following relation:

$$
\begin{equation*}
\frac{m_{1} v_{1 i}^{2}}{2}+\frac{m_{2} v_{2 i}^{2}}{2}=\frac{m_{1} v_{1 f}^{2}}{2}+\frac{m_{2} v_{2 f}^{2}}{2} \tag{3}
\end{equation*}
$$

Note, however, that in the case of an inelastic collision, the total momentum is conserved but total kinetic energy is certainly not since some (or possibly all) initial kinetic energy is dissipated into heat.

## Equipment:

5. LabQuest 2
6. Motion Detector
7. Track
8. Cart with Plunger / Cart Without Plunger

## Procedure:

In this lab, we will study three different kinds of collisions:

1. Elastic collision between two magnetic carts (rebound off one another)
2. Inelastic collision between two Velcro carts (stick together)
3. Inelastic collision between a magnetic cart and a Velcro cart
4. Data collection
a. Set motion detectors, one on each of the two ends of the track. Explore the effects of starting the experiment at different distances from the motion detectors and try to determine an optimal distance
b. Connect the motion detectors to channels 1 and 2 of the LabQuest device. Verify that your motion detectors are connected in the manner in which you intended.
c. Measure the total mass of each cart (which may include masses of bars to be placed on the cart). Name and record the cart masses as $m_{1}$ and $m_{2}$.
d. Practice gentle collisions between the carts: Velcro with Velcro, Magnetic with Magnetic and Velcro with Magnetic.
e. Pick two magnetic carts to formally begin the experiment. Press start on the LabQuest device, and then release the carts. Make sure your hands are not in the 'line of sight' of a motion detector.
f. Collect data and determine the instantaneous velocity (approximated by an average
velocity here) of each cart just before and just after the collision using the options you have used in previous labs in the "Analyze" tab. Do at least 5 runs for each collision and insert values in appropriate tables below or in your lab report.
g. Repeat step (e) and (f) for the other 2 types of collisions (Velcro with Velcro and Velcro with Magnetic).

Table 1. Magnetic with Magnetic
$\mathrm{m} 1: \quad \mathrm{m} 2:$

| Trial | $v_{1 i}$ | $v_{2 i}$ | $p_{i}$ <br> $=m_{1} v_{1 i}$ <br> $+m_{2} v_{2 i}$ | $K_{i}$ <br> $=K_{1 i}+K_{2 i}$ | $v_{1 f}$ | $v_{2 f}$ | $p_{f}$ <br> $=m_{1} v_{1 f}$ <br> $+m_{2} v_{2 f}$ | $K_{f}$ <br> $=K_{1 f}$ <br> $+K_{2 f}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |
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| 4 |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |

Table 2. Velcro with Velcro
$\mathrm{m} 1: \quad \mathrm{m} 2:$

| Trial | $v_{1 i}$ | $v_{2 i}$ | $p_{i}$ <br> $=m_{1} v_{1 i}$ <br> $+m_{2} v_{2 i}$ | $K_{i}$ <br> $=K_{1 i}+K_{2 i}$ | $v_{1 f}$ | $v_{2 f}$ | $p_{f}$ <br> $=m_{1} v_{1 f}$ <br> $+m_{2} v_{2 f}$ | $K_{f}$ <br> $=K_{1 f}$ <br> $+K_{2 f}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |

Table 3. Magnet with Velcro
$\mathrm{m} 1: \quad \mathrm{m} 2$ :

| Trial | $v_{1 i}$ | $v_{2 i}$ | $p_{i}$ <br> $=m_{1} v_{1 i}$ <br> $+m_{2} v_{2 i}$ | $K_{i}$ <br> $=K_{1 i}+K_{2 i}$ | $v_{1 f}$ | $v_{2 f}$ | $p_{f}$ <br> $=m_{1} v_{1 f}$ <br> $+m_{2} v_{2 f}$ | $K_{f}$ <br> $=K_{1 f}$ <br> $+K_{2 f}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |  |  |  |
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| 4 |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |

2. Analysis
3. Use the data collected above to calculate the total initial momentum, total final momentum, total initial kinetic energy, and total final kinetic energy of each type of collision for each run.

Magnetic with Magnetic:

Velcro with Velcro:

Magnetic with Velcro:
2. Verify that the total initial momentum equals the total final momentum for each collision in (1). If not, what is the \% difference?
3. Check that, for each type of collision, the total initial kinetic energy equals the total final kinetic energy. If not, what is the \% difference once again? Ideally, which of the three kinds of collisions carried out is reflective of an elastic collision and explain why the other two cannot be regarded as such?
4. In reality, even for the elastic collision, you should notice that the total initial and final momentum of the system differ. This is because friction acts on both carts, producing an impulse. Determine the impulse of the system for one collision from the experiment and compare to the calculated impulse. HINT: you'll need the coefficient of kinetic friction that you determined from the first part of this experiment.

## Torque and Static Equilibrium: Pre-lab

1) What is the goal of this experiment? Summarize in a few sentences.
2) Claim: to compute the experimental net torque, I don't need to multiply by g. Why?
3) How will you measure the lever arm for each weight?
4) Why should I balance the ruler at its center of mass?

## Torque and Static Equilibrium

## Objectives:

To gain an understanding for cases in which the net torque, as well as the net force, acting on a system of interest equals zero.

## Introduction:

A solid object is in static equilibrium, if the following conditions are satisfied:

$$
\begin{align*}
& \vec{F}_{\text {Net }}=\sum_{i=1}^{N} \vec{F}_{i}=0  \tag{1}\\
& \vec{\tau}_{\text {Net }}=\sum_{i=1}^{N} \vec{\tau}_{i}=0 \tag{2}
\end{align*}
$$

Equations (1) and (2) are really sets of three independent equations each in terms of their $x, y$, and $z$ component directions. The net force (1) expresses the net tendency for a system to move as a whole body (usually we speak about the motion of the center of mass) and the net torque (2) describes the net tendency for a system to spin about a point (an arbitrary reference point, usually the pivot point); so long as the system is balanced, both the net force and the net torque are zero.

Figure 1 shows a system, consisting of a rigid rod upon which three masses are balanced:


Figure 1
Each hanging weight contributes a torque about the pivot point (i.e., the fulcrum). Since the weights are perpendicular to their lever arms, the magnitude of the torque produced by any individual weight can be calculated as:

$$
\begin{equation*}
\tau=m g l \tag{3}
\end{equation*}
$$

where $l$ is the lever arm, which is the distance from the pivot point to the respective weight.

In the case shown above, we have:

$$
\begin{align*}
\tau_{1} & =m_{1} g l_{1},  \tag{4}\\
\tau_{2} & =m_{2} g l_{2},  \tag{5}\\
\tau_{2} & =m_{2} g l_{2} \tag{6}
\end{align*}
$$

It should be noted that torque is a vector $(\vec{\tau}=\vec{r} \times \vec{F})$, and as such, the direction of the torque must be accounted for; for example, a clockwise rotation yields a negative torque value.

## Equipment:

1. Meter stick
2. Meter stick clamps (knife edge)
3. Weights
4. Fulcrum (balance support)

## Procedure:

A. Data collection

1. Balance and secure the meter stick in the knife edge clamp.
2. Record the position of the balance point (center of mass) of the meter stick relative to the left end of the stick as $l_{0}$.
3. Experiment 1: Using the clamps, hang $\mathrm{m}_{1}=50 \mathrm{~g}$ on the right end of the meter stick and hang $\mathrm{m}_{2}=100 \mathrm{~g}$ on the other side. Do you need to include the mass of the clamps? Adjust the position of the masses by sliding them along the ruler until the system is balanced. Record these distances from the pivot point in Table 1.
4. Experiment 2: Place the masses used above on the right side of the balance point, one at 20 cm and the other at 30 cm , and now add a third mass on the left side (as in Figure 1). Slide the third mass around until the system balances. Do you need to include the mass of the clamps? Record the masses and lever arms in Table 2.

Center of mass of the meter stick: $l_{0}=$

Table 1

|  | Left side | Right side |
| :---: | :--- | :--- |
| Mass, m |  |  |
| Distance <br> (lever arm) |  |  |

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|  | Right side 1 | Right side 2 | Left side |
| :---: | :---: | :---: | :---: |
| Mass, m |  |  |  |
| Distance <br> (lever arm) |  |  |  |

B. Calculations

1. For Experiment 1, find the net torque of the system:
2. What is the theoretical lever arm for $m_{2}$ for Experiment 1 ? Use this value to find the percent error in the net torque.
3. For Experiment 2, find the net torque of the system:
4. What is the theoretical lever arm for the mass on the left side for experiment 2 ? Use this value to find the percent error in the net torque.

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## Discussion:

1. List several potential sources of error in the experiments.
2. Does the fulcrum contribute any torque to the system?
3. Explain what would happen if there was a net torque acting on the system.

## Oscillations and Simple Harmonic Motion: Pre-lab

1) What is the goal of this experiment? Summarize in a few sentences.
2) What is the difference between Part 1 and Part 2?
3) When I measure the displacement in Part 1, where should I start?
4) Why do I divide by 10 in Part 2?

## Oscillations and Simple Harmonic Motion

## Objectives:

To understand simple harmonic motion and measure the spring constant, $k$, of a spring via two different methods.

## Introduction:

Consider a block attached to a spring. The block can move only in one dimension, say along the $x$-axis. It has a mass $m$, and there is a spring force acting on it. The spring force is a variable force, and it is proportional to the amount by which the spring is stretched or contracted beyond its normal length, and it acts opposite in direction to the displacement of the block. This can be summed up in the equation

$$
F=k x
$$

where $k$ the spring constant, the proportionality of the force to displacement.
The units of the spring constant are usually Newtons/cm. The negative sign means that the force is a restoring one. Applying Newton's $2^{\text {nd }}$ Law, we have:

$$
F=m a=m \frac{d^{2} x}{d t^{2}}
$$

Therefore

$$
a=\frac{d^{2} x}{d t^{2}}=\frac{F}{m}=-\frac{k}{m} x
$$

Let us call

$$
\omega^{2}=\frac{k}{m}
$$

so that we have

$$
\frac{d^{2} x}{d t^{2}}=-\omega^{2} x
$$

We'd like to solve this equation for $x(t)$. This class of equations where the second derivative of $x$ is equal to some constant times $x$ has a simple solution. We can now write the solution for $x: x$ varies as the cosine of time:

$$
x(t)=A \cos t
$$

where $A$ is some constant called the amplitude. This solution assumes that the initial velocity of the object is 0 . As in all cosine functions, this will repeat itself, going through the same values over some specific time. This pattern will repeat itself forever.

We call this type of motion periodic. The time it takes the object to undergo one full oscillation where the motion will begin to repeat itself and return to the position from which it started is called the period. It is typically denoted by $T$. The amplitude $A$ is the maximum distance the mass gets away from the equilibrium, or unstretched position, which is also the starting point (try plugging in $t=0$, then $x=A$ ). The value of $x$ will go from $+A$ to $-A$ (just like cosine goes from +1 to -1 ). It takes the cosine function $360^{\circ}$ or $2 \pi$ radians to repeat. Therefore

$$
2 \pi=\omega T=\sqrt{\frac{k}{m}} T, \text { or } T=\frac{2 \pi}{\omega}=2 \pi \sqrt{\frac{m}{k}}
$$

As can be seen, the period is completely independent of the amplitude. You can start your oscillations from any position and the period should always be the same. Therefore, the spring equation has the same form as something called a simple harmonic oscillator.

It's important that you do not try to stretch a spring too far. If you do, then the spring will not return to its original length, which will change the spring constant of the spring. In other words, if you stretch it too much, you have broken the spring.

## Equipment:

1. Spring
2. Timer
3. Set of Weights

## Procedure:

Part 1. Measurement of the spring constant $k$ by use of weight.

If the spring is holding up a mass $m$ and is at equilibrium, the force of the spring equals the weight of the mass.

$$
k x=F_{\text {spring }}=\text { Weight }=m g
$$

As we increase the mass, the amount stretched will increase, but the spring constant should remain the same, for small stretching.

## Setup/Procedure

1. Hang the spring with a mass hanger (weight) on it from the stand. Measure the position of the bottom of the mass hanger. If the spring is conical in shape, hang it so the smaller diameter end is at the top. All of your measurements are taken with respect to this point.
2. Add 20 grams to the hanger and measure the new position of the bottom of the hanger in other words, how much the spring stretched.
3. Repeat 4 more times each time adding another 20 grams. Enter the relevant data in Table 1.

Table 1

| $m(\mathrm{~kg})$ | $x(\mathrm{~m})$ | $k=m g / x$ <br> $(\mathrm{~N} / \mathrm{m})$ | $\bar{k}$ (average in <br> units of N/m) | $\Delta k$ (root <br> mean square <br> error in <br> $\mathrm{N} / \mathrm{m})$ |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

Extra Credit: Use Excel to plot $F(=m g)$ vs $x$. (You can adjust the scale of the $x$-axis by double clicking on it.) Find the slope of the graph drawn (from the Chart menu, choose Add Trendline and highlight the option to display the equation). The slope should be the spring constant $k$. Check that your units are consistent. How does this result for $k$ compare with that for $\bar{k}$ in Table 1 ?

Part 2. Measurement of the spring constant $\boldsymbol{k}$ by use of oscillations.

## Setup/Procedure

Setup the mass/spring system with the 5 masses you used in Part 1. For each value of mass, measure the period. Do this by displacing the hanging weight from equilibrium by a small amount, letting go, subsequently timing 10 full oscillations, and then finally dividing this time by 10. Enter the relevant data in Table 2.

## Table 2

| $m(\mathrm{~kg})$ | $T(\mathrm{~s})$ | $k=4 \pi^{2} m / T^{2}$ <br> $(\mathrm{~N} / \mathrm{m})$ | $\bar{k}$ (average in <br> units of N/m) | $\Delta k$ (root <br> mean square <br> error in <br> $\mathrm{N} / \mathrm{m})$ |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Extra Credit: Plot your data of mass and period on Excel so that you get a straight line, so plot $T^{2}$ vs. $m$. Remember that $T=2 \pi \sqrt{m / k}$. From this straight-line graph, find the slope (as you did in Part 1) and deduce $k$. What are the units of $k$ ? Compare it with the value of $\bar{k}$ you found in Table 2.

## Discussion:

(1) Why isn't the mass of the spring included in the expressions for the spring constant?
(2) What is the uncertainty in your mass measurement, length measurement, and time measurement?
(3) From the uncertainties stated in (2), find the average uncertainty in the spring constant $k$ from Parts I and II of the lab. Recall the quadrature rule for adding fractional uncertainties.


[^0]:    ${ }^{1}$ Figure 2 taken from http://www.vernier.com/experiments/pwv/12/static_and_kinetic_friction/

